STAT 2290 Homework 7 Solutions

Exercise 1.

(Hypothesis testing for mean with known population standard deviation)
An LED lamp manufacturer guarantees that the mean life of a certain type of LED lamp is at least 25,000 hours. A random sample of 49 LED lamps has a mean life of 24,800 hours. Assume the population is normally distributed and the population standard deviation is 500 hours. At α = 0.05, do you have enough evidence to reject the manufacturer's claim?

Solution.
The manufacturer claims that the population mean is at least 25000 hours:

State Hypotheses:

- $H_0 : \mu \geq 25000$
- $H_a : \mu < 25000$

To do our tests, we must ensure methods learned in this class are valid:

Check conditions:

- 10% Condition: There are certainly at least $10n = 10 \cdot 49 = 490$ lamps made by this manufacturer. ✓
- Normal/Large Sample Condition: $n = 49 \geq 30$. ✓

Both conditions are met, so we may proceed.

Compute P-value:

$$\begin{aligned} P &= \text{pnorm}(24800, 25000, \frac{500}{\sqrt{49}}) \\ &= \text{pnorm}(24800, 25000, 71.4) \\ &= \text{pnorm}(\frac{24800 - 25000}{71.4}) \\ &= \text{pnorm}(-2.8) \\ &= 0.0025 \end{aligned}$$

Conclude:
Since $P = 0.0025 < \alpha = 0.05$, we reject H_0. There is convincing evidence that the mean life of such LED lamp is less than 25,000 hours.

Exercise 2.

(Hypothesis testing for mean with known population standard deviation)

A cookie manufacturer claims that the mean sugar content in each of the cookies produced is no more than 18%. A random sample of 56 cookies has a mean sugar content of 19%. Assume the population standard deviation is 4%. At α = 0.02, do you have enough evidence to reject the manufacturer's claim?

Solution.
This problem looks like the proportion problem, but it is actually a mean problem since we are computing the sugar level of each cookie, rather than the proportion of cookies that have a certain sugar level.

State Hypotheses:

- $H_0 : \mu \leq 0.18$

- $H_a : \mu > 0.18.$

Check conditions:

- 10% Condition: There are certainly at least $10n = 10 \cdot 56 = 560$ cookies made by this manufacturer. ✓
- Normal/Large Sample Condition: $n = 56 \geq 30$. ✓

Both conditions are met, so we may proceed.

Compute P-value:

$$\begin{aligned} P &= \text{pnorm}(0.19, 0.18, \frac{0.04}{\sqrt{56}}, \text{lower.tail=FALSE}) \\ &= \text{pnorm}(\frac{0.19 - 0.18}{0.04/\sqrt{56}}, \text{lower.tail=FALSE}) \\ &= \text{pnorm}(1.87, \text{lower.tail=FALSE}) \\ &= 0.03 \end{aligned}$$

Conclude:
Since $P = 0.003 > \alpha = 0.02$, we fail to reject H_0. There is **not enough** convincing evidence that the mean sugar content in each of the cookies produced is more than 18%.

Exercise 3.

(Hypothesis testing for mean with unknown population standard deviation)

It is generally accepted that the mean body temperature of an adult human is 98.6°F. However a recent study finds that in a random sample of 130 adult humans, the average temperature of the subjects is 98.25 with a standard deviation of 0.73. Does this recent study provide convincing evidence against the conventional wisdom that $\mu = 98.6^\circ F$?

Solution.
Since the problem did not state an α, let's set a stricter significance level of $\alpha = 0.01$ before we proceed.

State Hypotheses:

- $H_0 : \mu = 98.6$
- $H_a : \mu \neq 98.6$

Check conditions:

- 10% Condition: There are certainly at least $10n = 10 \cdot 130 = 1300$ adult humans in the population. ✓
- Normal/Large Sample Condition: $n = 130 \geq 30$. ✓

Both conditions are met, so we may proceed.

Compute P-value:

$$\begin{aligned} P &= 2 \cdot \text{pt}\left(\frac{98.25 - 98.6}{0.73/\sqrt{130}}, \text{df} = (130 - 1)\right) \\ &= 2 \cdot \text{pt}(-5.4, \text{df} = 129) \\ &= 2.3 \times 10^{-7} \end{aligned}$$

Conclude:
Since $P = 2.3 \times 10^{-7} < \alpha = 0.01$, we reject H_0. There is convincing evidence that the mean body temperature of an adult human is not 98.6°F.

Note: There is some evidence that the average normal human temperature is *declining* with each decade.

Exercise 4.

(Test for proportions)

People of taste are supposed to prefer fresh-brewed coffee to the instant variety. On the other hand, perhaps many coffee drinkers just want their caffeine fix. A skeptic claims that only half of all coffee drinkers prefer fresh-brewed coffee. To test this claim, we ask a random sample of 50 coffee drinkers in a small city to take part in a study. Each person tastes two unmarked cups— one containing instant coffee and one containing fresh-brewed coffee—and says which he or she prefers. We find that 36 of the 50 choose the fresh coffee. Do these results give convincing evidence that coffee drinkers favor fresh-brewed over instant coffee?

Added in solution: Use $\alpha = 0.01$ as the significance level.

Solution.
We interpret the skeptic's claim of "only half" to mean "at most half" or at most 50%.

State Hypotheses:

- $H_0 : p \leq 0.5$
- $H_a : p > 0.5$

Check conditions:

- 10% Condition: There are certainly at least $10n = 10 \cdot 50 = 500$ coffee drinkers in the population. ✓
- Large Counts Condition: $np = 50 \times 0.5 = 25 \geq 10$ and $n(1-p) = 50 \times 0.5 = 25 \geq 10$. ✓

Both conditions are met, so we may proceed.

Compute P-value:

$$\begin{aligned} P &= \text{pnorm}(\frac{36}{50}, 0.5, \sqrt{\frac{0.5 \times 0.5}{50}}, \text{lower.tail=FALSE}) \\ &= \text{pnorm}(0.72, 0.5, 0.07, \text{lower.tail=FALSE}) \\ &= \text{pnorm}(\frac{0.72 - 0.5}{0.07}, \text{lower.tail=FALSE}) \\ &= \text{pnorm}(3.11, \text{lower.tail=FALSE}) \\ &= 0.0009 \end{aligned}$$

Conclude:
Since $P = 0.0009 < \alpha = 0.01$, we reject H_0. There is convincing evidence that more than 50% of coffee drinkers prefer the instant variety.